



SRI AKILANDESWARI WOMEN COLLEGE WANDIWASH

LINEAR PROGRAMMING

CLASS:III UG MATHEMATICS

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INTRODUCTION

The origin of Transportation Models dates back to 1941 when F.L Hitchcock presented a study entitled “The distribution of a product from several sources to numerous localities”. The presentation is regarded as the first important contribution to the solution of transportation problem.

Transportation model was first introduced by F.L. Hitchcock in 1941. Later on, it was further improved by T.C. Koopman in 1949 and G.B. Dantzig in 1951. The objective of transportation is to transport the similar quantities which are initially stored at various origins (supply) to different destinations (demand) in such a way that the total transportation cost is minimum .

The object is to minimize the cost of transportation while meeting the requirement at the destinations. Transportation problem may also involve movement of a product from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers

DEFINITION OF TRANSPORTATION PROBLEM:

The Transportation problem is a special type of linear programming problem where the objective is to minimize the cost of distributing a product from a number of **sources** or **origins** to a number of **destinations**. Because of its special structure, the usual simplex method is not suitable for solving transportation problems. These problems require a special method of solution.

BASIC DEFINITIONS:

FEASIBLE SOLUTION:

A set of non-negative values X_{ij} , $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$ that satisfies the constraints (rim conditions) is called **feasible solution**.

BASIC FEASIBLE SOLUTION:

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $m+n-1$ non-negative independent allocations is called a **basic feasible solution (BFS)** to the transportation problem.

NON-DEGENERATE BASIC FEASIBLE SOLUTION:

A basic feasible solution to a $(m \times n)$ transportation problem is said to be **non - degenerate basic feasible solution** if it contains exactly $m+n-1$ non-negative allocation in independent positions.

DEGENERATE BASIC FEASIBLE SOLUTION:

A basic feasible solution that contains less than $m+n-1$ non- negative allocations is said to be **degenerate basic feasible solution**.

OPTIMAL SOLUTION:

A feasible solution (not necessarily basic) is said to be an **optimal solution** if it minimizes the total transportation cost. The number of basic variables in an $m \times n$ a balance transportation problem is at the most $m+n-1$.

TYPES OF TRANSPORTATION PROBLEM IN OPERATION RESEARCH:

- ❖ Balanced transportation problem.
- ❖ Unbalanced transportation problem.

TYPES OF TRANSPORTATION PROBLEM IN OPERATION RESEARCH:

BALANCE TRANSPORTATION PROBLEM:

The Transportation problem in which the total supply and demand are equal is called balanced transportation problem.

$$(i.e)., \sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

UNBALANCED TRANSPORTATION PROBLEM:

The Transportation problem in which the total supply and demand are not Equal is called unbalanced transportation problem.

$$(i.e)., \sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$$

APPLICATION OF TRANSPORTATION PROBLEM :

- Minimize shipping costs from factories to warehouses (or from warehouses to retail outlets).
- Determine lowest cost location for new factory, warehouse, office or other outlet facility.
- The quantity of units delivered directly correlates to the cost of shipping from a source to a destination.
- Where cost is not a factor, it is employed for transportation to fulfill such responsibility.

AIM OF TRANSPORTATION PROBLEM:

- To find out the optimum transportation schedule keeping in mind cost of Transportation to be minimized.
- The origin of a transportation problem is the location from which shipments are dispatched.
- The destination of a transportation problem is the location to which shipments are transported.
- The unit transportation cost is the cost of transporting one unit of the consignment from an origin to a destination

METHODS OF FINDING AN INITIAL BASIC FEASIBLE SOLUTION:

1. NORTH-WEST CORNER METHOD.
2. LEAST COST ENTRY METHOD.
3. ROW MINIMA METHOD.
4. COLUMN MINIMA METHODSD.
5. VOGEL'S APPROXIMATION METHOD.

PROPERTIES OF A BASIC FEASIBLE SOLUTION OF A TRANSPORTATION PROBLEM:

A basic feasible solution of an $m \times n$ transportation problem is said to be non-degenerate, if it has the following two properties:

- Starting basic feasible solution must contain exactly $(m+n-1)$ number of individual allocations.
- These allocations must be in independent position.

CONDITIONS FOR INITIAL BASIC FEASIBLE SOLUTION:

- ❖ By feasible solution we mean a set non-negative individual allocations ($X_{ij} \geq 0$) which satisfied the row and column conditions (rim condition).
- ❖ A feasible solution is said to be basic if the number of positive allocations equals $m+n-1$. That is one less than the number of rows and columns in a transportation problem.

DEFINITION OF NORTH-WEST CORNER RULE:

The north-west corner rule is a method used to calculate an initial feasible solution for a transportation problem. In this technique we start by selecting the basic variables from the top left cell also known as the north-west corner cell.

ADVANTAGE OF NORTH-WEST CORNER METHOD:

- ❖ This method is very effective as it is provided by step by step solution
- ❖ It allows for quick calculations for large-scale problem

DISADVANTAGE OF NORTH-WEST CORNER METHOD:

- ❖ Takes more time in obtaining optimal solution.

DEFINITION OF LEAST COST METHOD :

The objective is to minimize the total transportation cost, we must try to transport as much as possible through those routes (cells) where the unit transportation cost is lowest. This method takes into account the minimum unit cost of transportation for obtaining initial solution and can be summarized as follows:

ADVANTAGE OF LEAST COST ENTRY METHOD:

- ❖ This method provides allocate solution as transportation cost is consider while making allocation.
- ❖ It is very simple and easy to calculate optimum solution under this method.

DISADVANTAGE OF LEAST COST ENTRY METHOD:

- ❖ This method is based on the selection through personal observation when there is tie in the minimum cost it does not follow any system tie rule.

DEFINITION OF ROW MINIMA METHOD:

This method consists in allocating as much as possible in the lowest cost cell of the first row so that either the capacity of the first plant is exhausted or the requirement at j^{th} distribution centre is satisfied or both.

USES OF ROW MINIMA METHOD:

- ❖ In the row minima method, the first row that is the lowest cost cell is exhausted.
- ❖ Our aim will be to allocate the Maximum either at the first source or demand at the destination or to satisfy both.
- ❖ Choosing the lowest element in the row and deducting it from all the elements.

**OBTAIN AN INITIAL BASIC FEASIBLE SOLUTION TO THE FOLLOWING
TRANSPORTATION PROBLEM USING LEAST COST METHOD.**

	1	2	3	4	SUPPLY
A	7	3	5	5	35
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
DEMAND	21	25	17	17	80

SOLUTION:

The given problem is unbalanced transportation .

Applying least cost method.

	1	2	3	4	5	SUPPLY
A	7 (6)	3 (17)	5 (11)	5 (1)	0	35/34/28/11/0
B	5 (15)	5	7	6	0	15/0
C	8 (6)	6	6	5 (6)	0	12/6/0
D	6	1 (19)	6	4	0	19/0
DEMAND	21/6/0	25	17/0	17/6/0	1/0	81

$$\begin{aligned}\text{Feasible Solution} &= m+n-1 \\ &= 4+5-1 \\ &= 8\end{aligned}$$

$$\text{No. of allocation} = 8$$

$$\text{No. of allocation} = m+n-1$$

This is non-degeneracy

$$\begin{aligned}\text{Transportation cost} &= (3 \times 6) + (5 \times 17) + (5 \times 11) + (0 \times 1) + (5 \times 15) + \\ &\quad (8 \times 6) + (5 \times 6) + (1 \times 19) \\ &= 18 + 85 + 55 + 0 + 75 + 48 + 30 + 19\end{aligned}$$

$$\text{Transportation cost} = 330$$

DEFINITION OF VOGEL'S APPROXIMATION METHOD:

Vogel's Approximation method is a heuristic method and is preferred to the methods described above. In the Transportation matrix if an allocation is made in the second lowest cell instead of the lowest, then this allocation will have associated with it a penalty corresponding to the difference of these two costs due to "loss of advantage". Vogel's Approximation method therefore makes effective use of the cost information and yields a better initial solution than obtained by the other methods.

APPLICATION OF VOGEL'S APPROXIMATION METHOD:

- ❖ Vogel's approximation method (VAM) is one of the methods use to calculate the initial feasible solution to a transportation problem.
- ❖ However , VAM is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost.

DEFINITION OF COLUMN MINIMA METHOD :

It is begin with the first column and allocate gradually moving towards the lowest cost cell of the column. This system is continued until the first destination centre is satisfied or the capacity of the second is exhausted, or both happens then, is called a column minima method.

USES OF COLUMN MINIMA METHOD:

- ❖ Column minima method involves choosing the lowest element and deducting it from all the other elements in the column.
- ❖ Begin with the first column and allocates gradually moving towards the lowest cost cell of the column.
- ❖ Allocate the minimum number (transportation cost) starting from 1st column of matrix.

ADVANTAGE OF VOGEL'S APPROMIXATION METHOD:

- ❖ The solution is close to the optimum
- ❖ The computing time is low because there are no complex matrix operation.
- ❖ Valid integer solutions are quickly found.
- ❖ Quickly done by hand-if the complexity allow it.

DISADVANTAGE OF VOGEL'S APPROMIXATION METHOD:

- ❖ The solution is not the optimum.
- ❖ The algorithm can hardly include fixed costs and multiple product cases.
- ❖ Nowadays, additional computing power is required for complex problem.

METHODS FOR OPTIMALITY TEST OF THE TRANSPORTATION PROBLEM:

Once an initial solution has been found, the next step is to test that solution for optimality. The following two methods are widely used for testing the solutions:

1. Stepping stone method
2. Modified Distribution Method (MODI). The two methods differ in their computational approach but give exactly the same results and use same testing procedure.

DEFINITION OF STEPPING – STONE METHOD:

The stepping stone method is an iterative technique for moving from an initial feasible solution to an optimal feasible solution.

DEFINITION OF MODIFIED DISTRIBUTION METHOD (MODI):

The Modified distribution method, cell evaluations of all the unoccupied cells are calculated simultaneously and only one closed path for the most negative cell is traced. Thus it provides considerable time saving over the stepping stone method. It provides a new means of finding the unused route with the largest negative improvement index. It can often provide considerable time savings over the stepping stone method for solving transportation problem.

APPLICATION OF MODI METHOD

- The modified distribution method provides a minimum cost solution to the transportation problem.
- MODI method is an improvement over stepping stone method.

USES OF MODI METHOD:

- The MODI (modified distribution) method is used to find the optimal solution when the initial basic solution is obtained.
- The modi method is an efficient method of checking the optimality of the initial feasible solution.

COMPARISON BETWEEN THE THREE METHODS:

- Northwest corner method is used when the purpose of completing demand and then the next and is used when the purpose of completing the supply and then the next. Advantage of northwest corner method is quick solution because computations take short time but yields a bad solution because it is very far from optimal solution.
- Vogel's approximation method and Least cost method are used to obtain the shortest route. Advantage of Vogel's approximation method and least cost method yields the best starting basic solution because gives initial solution near to optimal solution but the solution of Vogel's approximation methods is slow because computations take long time. The cost of transportation with Vogel's approximation method and least cost method is less than northwest corner method.

CONCLUSION:

In conclusion, transportation problem approach helps to solve most is the real life transportation problems with multi objective and imprecise and precise parameters through an interactive decision making process. This work aims to present an interactive possibilistic linear programming problem approach for solving multi objective transportation problems with imprecise cost, demand and supply. By this approach simultaneously the most possible value of the imprecise total costs are minimized, possibility of obtaining lower total costs are maximized and, the risk of obtaining higher total costs are minimized. Transportation problem is special kind of linear programming problem. Because of the transportation problem special structure the simple method is not suitable. But which may be utilized to make efficient computational techniques for its solution. Generally transportation problem has a number of origin and destination

THANK YOU

